

Theoretical calculation on trappable tidal current energy in tidal channel

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Abstract. Based on the principle of energy conservation, a rigorous mathematical assessment is built on the prediction of tidal current power in an open channel. According to the mathematical derivation results, the maximum trappable rate of tidal power is mainly determined by the channel geographic properties, such as Manning roughness coefficient at channel boundaries, channel length and average water depth, along with the power plant energy extraction efficiency, which are fully related to the terrain and surrounding environment.

Key words. Tidal current energy, theoretical calculation, installed capacity.

1. Introduction

The power of tidal current and stream energy relies on the kinetic energy lead by the potential of water depth difference between entrance and exit of water channel. Accurate evaluation of reserve and precise assessment of energy exploitation are essential for the success of site selection and power generation. The classical ways of extractable power prediction are developed based on the results from sufficient amount of numerical simulations by the research and development teams in Europe and USA, such as Energy Technology Support Unit (ETSU) [1], Black & Veatch Company [2,3], and JOULE II Non-Nuclear Energy Program by European commission DG, XII [4].

There are a few method developed for the resources prediction. According to the previous work of Lv and Qiao [5] where the methods of predicting tidal current energy were reviewed, the methods were developed in two ways based on the concept of energy flux and dynamics of tidal current kinetic motion equation respectively.

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The former one can be further divided into Farm and Flux method and has a wide range of application. The latter one was proposed by Garrett and Cummins [6], and suitable for the cases with a constricted channel in which the tides at both ends were assumed to be unaffected by the currents through the channel. Known as ??device arraying method??. Farm method assumes that there are a series of power units installed in tidal channel, and the total generated power is dependent with the efficiency of power units. However, it is found that Farm method overestimate exploitation amount of tidal current energy according to Black & Veatch [2,3]. Flux method takes tidal current flux and effective impact factor (SIF) into consideration, and calculates the exploitation power as a result of multiplication of SIF and cross-section energy flux in channel. The calculation results by Flux and Farm methods on extractable tidal current energy are far from consistent, and it is difficult to confirm which one is more accurate.

2. Theoretical Derivation

2.1. Calculation of tidal current energy in empty channel

In the present study, we assume the flow velocity is mainly lead by the different water depth between the entrance and exit of the channel, which means that all the kinetic energy of currents passing through the channel is induced by the potential of head loss of two ends of channel. In an empty water channel with no generators installed, energy loss is merely caused by the friction and viscous resistance. Hence the average cross section flow velocity in the channel can be predicted as in Eq. 1, with head loss, h_f , channel bed manning roughness coefficient, n , channel length, l , and average water depth, H , where H includes average water depth, h , and average surface elevation, ζ .

$$v_c = \frac{1}{n} H^{\frac{2}{3}}, \quad (1)$$

The power contained in the flow passing through the channel, P_t , is shown in Eq. 2,

$$P_t = \frac{1}{2} A \rho v_c^3, \quad (2)$$

where ρ is the water density and A is average cross section area. Substituting v_c in Eq. 2 by Eq. 1, the power can be presented in Eq. 3.

$$P_t = \frac{A \rho H^2}{2n^3} \left(\frac{h_f}{l} \right)^{3/2} \quad (3)$$

2.2. Calculation of installed capacity of tidal current energy

The power of flow stream in the empty channel, P_t , can be regarded as the maximum available tidal current energy in theory. However, when the power plants

are installed, the power of flow stream should be less than Pt due to the reasons that the devices block the channel leading slower flow velocity and smaller water flux.

The potential tidal stream energy rate in a channel with installed power plants is given in Eq. 4, by the head loss, h_f , gravity acceleration, g , and flow flux, Q .

$$E_p = \rho g Q h_f \quad (4)$$

The flow flux (Q) is a variable obtained from the average flow velocity behind the power plants (v_e) and average cross section area (A), i.e., $Q = v_e A$.

The kinetic energy rate of the flow behind the power plants, E_k , is hence calculated by Eq. 5.

$$E_k = \frac{1}{2} (\rho Q) v_e^2 = \frac{\rho Q^3}{2A^2} \quad (5)$$

Consequently, the total energy loss rate, E_{ch} , consisting one part extracted by the devices and the other part consumed by the friction and flow viscosity, can be obtained from Eq. 6.

$$E_{ch} = E_p - E_k = \rho g Q h_f - \frac{\rho Q^3}{2A^2} = \rho Q \left(g h_f - \frac{Q^2}{2A^2} \right) \quad (6)$$

The efficiency coefficient of the power generators is assumed to be k , and then the total extractable power can be calculated as in Eq. 7. It is noticed that the efficiency coefficient of well-designed power plants can reach up to approximately 80%.

$$P = k E_{ch} = k \rho Q \left(g h_f - \frac{Q^2}{2A^2} \right) \quad (7)$$

At a specific current channel, both h_f and A can be obtained from site measurement, and k can be estimated from the available generators. Assuming all the three parameters are constants, P can be seen as a function of Q from Eq. 7. Based on Fermat's theorem, the maximum P occurs when the first-order derivative of Q is zero, as shown in Eq. 8, under the condition of an optimized flow flux, Q_m , as indicated in Eq. 9. The flow flux actually represents the average wake velocity, it should be notice that the interaction between the power generator devices and flow is not considered in this study. Whereas it is interesting to observe from Eq. 9 that the optimal flow flux in current channel is dependent on the water head and average cross section area only, but independent on the energy extraction efficiency of power plants. Hence the maximum extractable power can be predicted and shown in Eq. 10.

$$\frac{dP}{dQ} = \frac{d}{dQ} \left[k \rho Q \left(g h_f - \frac{Q^2}{2A^2} \right) \right] = 0 \quad (8)$$

$$E_k = \frac{1}{2} (\rho Q) v_e^2 = \frac{\rho Q^3}{2A^2} \quad (9)$$

$$\tau = \tau_0 (1/2 - \xi) \quad (10)$$

The maximum trappable power rate of tidal current energy can be calculated in Eq. 11., and determined only by the channel properties of manning roughness coefficient, channel length and average water depth, along with the power plant energy extraction efficiency.

$$N = \frac{P_m}{P_t} P_t \quad (11)$$

3. Conclusion

This paper offers a rigorous numerical assessment on the prediction of tidal current power in open channel. The method is based on the principle of energy conservation, and the formulas are developed according to the relation between total cross-section power and potential and kinetic energy. It shows that the maximum trappable rate of tidal power is mainly determined by the channel properties, such as manning roughness coefficient, channel length and average water depth, along with the power plant energy extraction efficiency, which are fully related to the terrain and surrounding environment. It also shows that the maximum trappable tidal current power is proportional to the total power in the channel.

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